RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. MID SEMESTER EXAMINATION, SEPTEMBER 2012

FIRST YEAR

Date : 10/09/2012 — **MATHEMATICS (Honours)**

Time : 11 am – 1 pm — Paper : I — Full Marks : 50

[Use separate answer-books for each group]

Group–A

1. Answer **any three** questions: 3x4
   a) State and prove the fundamental theorem on equivalence relations. 4
   b) Let S be a non-empty finite set. Prove that a mapping $f: S \to S$ is injective if and only if it is surjective. 4
   c) Let S be a finite set with n elements. Find the number of symmetric relations on S. 4
   d) Describe the smallest equivalence relation on $\mathbb{R}$ containing the line $x-y=1$ in (x, y) plane. 4
   e) Check whether the following statements are true for $A, B, C \subseteq U$, U being the universal set: 4
   (i) If $A \bigcup B = A \bigcup C$, then $B = C$
   (ii) If $A \bigcap B^c = A \bigcap C^c$, then $B = C$
   (iii) If $A \Delta B = A \Delta C$, then $B = C$
   (iv) IF $A \times B = A \times C$, then $B = C$.
   (Justify your claim in each case)

2. Answer **any two** questions: 2x3
   a) State the principle of mathematical induction and use it to prove that $(n+1)! > 2^n$ for all $n \geq 2$. 1+2
   b) State the lub axiom for subsets of $\mathbb{R}$. Prove that $\{A : A \subseteq \mathbb{R}$ and Sup $A = 0\}$ is an infinite set. 1+2
   c) Let $X = [0,1]$, $Y = [\sqrt{2}, \sqrt{3}] \bigcap Q$ and T be the set of all functions from X to Y i.e. $T = \{f: f$ is a function from X to $Y\}$.
   Find sup A and inf A where $A = \{ f(x) : f \in T, x \in X\}$.

Answer **any one** question out of 3 and 4: 1x7

3. a) Prove that every convergent sequence in $\mathbb{R}$ is bounded. Is the converse true? Justify. 2½+1½
   b) Prove that the sequence $\{u_n\}_{n\in\mathbb{N}}$ defined by $0 < u_1 < u_2$ and $u_{n+2} = \frac{2u_{n+1} + u_n}{3}$ for $n \geq 1$, converges to $\frac{u_1 + 3u_2}{4}$. 3

4. a) Prove that a monotone increasing sequence in $\mathbb{R}$, which is bounded above, converges to its supremum. 3
   b) Prove that the seaquence $\{u_n\}_{n\in\mathbb{N}}$ defined by $u_1 = \sqrt{6}$ and $u_{n+1} = \sqrt{6 + u_n}$ for $n \geq 1$, converges to 3. 4

Group–B

5. Answer **any three** questions: 3x4

a) Show that the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is right angled if $(a+b)(al^2+2hlm+bmh^2)=0$. 4

b) Show that the equation of the line joining the feet of the perpendiculars from the point $(d,0)$ on the lines $ax^2+2hxy+by^2=0$ is $(a-b)x+2hy+bd=0$. 4

c) If the tangents at P, Q of a parabola meet at a point T and S be the focus, then prove that $ST^2=SP.SQ$. 4

d) Show that the equation of the tangent to the conic $\frac{l}{r}=1+e\cos\theta$ parallel to the tangent at $\theta=\alpha$ is given by $l(e^2+2e\cos\alpha+1)=r(e^2-1)[\cos(\theta-\alpha)+e\cos\theta]$. 4

e) Reduce the equation $x^2-4xy+4y^2+2x-4y+c=0$ to its canonical form and find its nature for different values of c. 4

6. Answer **any one** question: 1x13

a) Solve the equation $\frac{dy}{dx}+Py=Qy^n$ in which P and Q are functions of x alone or constants. 4

b) Find an integrating factor of the equation $(y^2+2x^2y)dx+(2x^3-xy)dy=0$ and hence solve it. 5

c) If $\frac{dr}{d\theta}+2r\tan\theta=\sin\theta$ and $r=0$ when $\theta=\frac{\pi}{3}$, find the maximum value of r. 4

7. a) Prove that the necessary and sufficient condition that $Mdx+Ndy=0$ be exact is $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$. 5

b) Solve: $(x^3+xy^4)dx+2y^3dy=0$. 4

c) Find the differential equation of $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, where λ is the only parameter. 4

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